Why qualifications at the Olympics?

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Abstract:

The optimal contest architecture for symmetric imperfectly discriminating contests is shown to be generically the two-stage tournament (rather than the one-stage contest). In the first stage the contestants compete in several parallel divisions for the right to participate in the second stage. In the second stage the short-listed finalists compete for the prize. Given a sufficient number of contestants, the two-stage tournament is either strictly better or at least as good as the one-stage contest for maximizing an individual's effort, for maximizing the aggregate effort and for minimizing the standard deviation of effort. For maximizing an individual's effort it is generally optimal to have only two finalists in the second stage. For maximizing the aggregate effort or minimizing the standard deviation of effort the optimal number of finalists in the second stage depends on the discriminating power of the contest success function.

Keywords: symmetric contest, imperfectly discriminating contest, logit, asymmetric equilibria, contest architecture, sport

JEL Classification codes: C72, L13, L83

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1. Introduction

A contest is a competition where the contestants simultaneously contribute effort to win a prize. In perfectly discriminating contests (*e.g.* Moldovanu and Sela 2001) the highest contributed effort secures a win (like in an all-pay auction). In imperfectly discriminating contests (*e.g.* Dixit 1987) the highest contributed effort has the highest probability of a win but it does not necessarily secure a win. Imperfectly discriminating contests are extensively used to study sport competitions (*e.g.* Szymanski 2003), political rent-seeking (*e.g.* Nitzan 1991, 1994), research and development and patent races (*e.g.* Nti 1997), labor incentives (e.g. Rosen 1986) etc.

In this paper I study the architecture of symmetric imperfectly discriminating contests. Casual evidence suggests that the one-stage contest is rarely organized (*e.g.* Amegashie 1999). Common practice instead is a two-stage tournament. In the first stage the contestants are grouped into several divisions where they compete for the right to participate in the second stage. In the second stage the contestants who won the first stage compete for the final prize. I will examine whether such a two-stage tournament is beneficial for the organizers who wish to maximize an individual's effort, to maximize the aggregate effort or to minimize the standard deviation of contributed effort. This paper is related to Gradstein and Konrad (1999) and Amegashie (1999) who studied the optimal architecture of symmetric imperfectly discriminating contests when the organizers want to maximize or minimize the aggregate (rent-seeking) effort. Moldovanu and Sela (2004) analyzed the optimal architecture of asymmetric perfectly discriminating contests. However, all above mentioned papers considered only a symmetric Nash equilibrium. In this paper I analyze the complete structure of equilibria including asymmetric equilibria (*e.g.* Perez-Castrillo and Verdier 1992).

The remainder of this paper is organized as follows. The one-stage contest (so called Tullock (1980) contest) is described in section two. The two-stage tournament is described in section three. These two contest architectures are examined in section four according to three criteria: maximum individual effort, maximum aggregate effort and minimum standard deviation of effort. Section five concludes.

2. One-stage contest

Consider $n \geq 2$ identical contestants for a prize $V > 0$. Each contestant contributes an effort $e_i \geq 0, i \in [1, n]$, and has a probability $p(e_i) = e_i^r \Big/ \sum_{j=1}^{n} e_j^r$ of winning the prize.[1] The logit form of the contest success function $p : \mathbb{R}_+ \rightarrow [0,1]$ was introduced by Tullock (1980) and subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). $r > 0$ is the discriminating power of the contest success function. When $r \rightarrow +\infty$ the imperfectly discriminating contest converges to the perfectly discriminating one. In the standard Tullock (1980) contest the contestants are assumed to be risk neutral with linear cost of effort. Kai and Schlesinger (1997) studied imperfectly discriminating contests with risk averse contestants and found that risk aversion has an indeterminate effect on the contributed effort. Moldovanu and Sela (2004) studied perfectly discriminating contests with convex cost of effort and found that the benefits of two-stage tournaments increase in the degree of convexity of a cost function.

Each contestant maximizes the net expected value of his or her contributed effort (1).

$$e_i = \arg\max_{e_i \geq 0} \left\{ \frac{e_i^r}{\sum_{j=1}^{n} e_j^r} V - e_i \right\}, i \in [1, n] \qquad (1)$$

[1] The technology that translates an individual's effort into the probability of winning is called the contest success function (*e.g.* Szymanski 2003).

The solution of problem (1) is presented in the second column of table 1 (*e.g.* Blavatskyy 2004, Perez-Castrillo and Verdier 1992). Depending on its discriminating power the one-stage contest may have a symmetric Nash equilibium (the second row of table 1), an asymmetric Nash equilibrium (the third row of table 1) or a Stackelberg equilibrium (the fourth row of table 1). Perez-Castrillo and Verdier (1992) and Cleeton (1989) discussed the asymmetric Nash equilibria in the Tullock (1980) contest. Baye and Shin (1999) and Perez-Castrillo and Verdier (1992) analyzed the Stackelberg equilibrium in the Tullock (1980) contest.

The intuitive explanation of equilibrium structure from table 1 is as follows. In poorly discriminating contests (when $r \leq n/(n-1)$) all contestants actively participate in the contest (*i.e.* they contribute positive effort) because there is a possibility to win a prize (almost by chance) while contributing low effort. In moderately discriminating contests (when $n/(n-1) < r \leq 2$) up to $n-2$ contestants may drop out of competition. They become passive contestants who contribute zero effort. The remaining active contestants are expected to contribute such a high effort that it erases any expectations of winning a prize for the passive contestants. In highly discriminating contests (when $r > 2$) only one contestant is active and $n-1$ contestants are passive.

Discriminating power of the contest success function	Individual effort	Aggregate effort	Standard deviation of effort
$0 < r \leq \frac{n}{n-1}$	$e_i = rV\frac{n-1}{n^2}$, $i \in [1, n]$	$\bar{e} = rV\frac{n-1}{n}$	0
$\frac{m+1}{m} < r \leq \frac{m}{m-1}$, $m \in [2, n-1]$	$e_i = rV\frac{m-1}{m^2}$, $i \in [1, m]$, $e_i = 0$, $i \in [m+1, n]$	$\bar{e} = rV\frac{m-1}{m}$	$rV\frac{m-1}{m}\sqrt{1/m - 1/n}$
$r > 2$	$e_1 = V(r-1)^{1-1/r}/r$, $e_i = 0$, $i \in [2, n]$	$\bar{e} = V(r-1)^{1-1/r}/r$	$V\frac{(r-1)^{1-1/r}}{r}\sqrt{1-1/n}$

Table 1 An individual's effort, the aggregate effort and the standard deviation of effort in one-stage contest with *n* contestants for a single prize *V*

3. Two-stage tournament

In the first stage of the tournament n contestants are grouped into $f \in [2, n/2]$ divisions with n/f contestants in each division. A separate contest is then organized in each division for the right to participate in the second stage. A contestant who won the first-stage contest proceeds to the second stage. All other contestants are eliminated from further competition. In the second stage f contestants (each of whom has won the first-stage contest) compete for the prize V. The second stage is thus equivalent to the one-stage contest described in section 2 with $n = f$.

Let $k \in [1, f]$ be the number of active contestants in the second-stage contest each of whom contributes an effort e_k. For each contestant who actively (passively) participates in the second stage, the net expected value of his or her contributed effort is $V/k - e_k$ (zero). Since there are k active contestants and $f - k$ passive contestants, the net expected value of the participation in the second stage is *ex ante* equal to $V_1 = k(V/k - e_k)/f = (V - ke_k)/f$. Therefore, the right for participation in the second stage yields an *ex ante* payoff V_1. The first-stage divisional contest is then equivalent to the one-stage contest described in section two with n/f contestants and a prize V_1 (*e.g.* Amegashie 1999).

4. Comparative statics

The contests' organizers (here onwards the organizers) may have three different objectives: to maximize an individual's effort, to maximize the aggregate effort, or to minimize the standard deviation of the contestants' effort. In other words, the organizers may wish to observe the highest winning effort (the breaking of a world record), to maintain the overall quality of the contest, or to foster a close contest (a competitive balance) resulting in a thrilling competition (*e.g.* Szymanski 2003, p. 1143).

4.1. Maximum individual effort

Suppose that the organizers want to maximize an individual's effort contributed in the two-stage tournament. In poorly discriminating contests (when $r \le n/(n-1)$) all contestants actively participate in the first stage and all short-listed contestants actively participate in the second stage ($k = f$). Consequently, the symmetric Nash equilibrium exists in both stages. In the second stage each contestant contributes effort $e_f = rV(f-1)/f^2$, which follows from the second row of table 1. In the first stage each contestant contributes effort $r\left(\frac{V}{f} - e_f\right)\frac{n/f-1}{(n/f)^2} = rV\left(1 - r + \frac{r}{f}\right)\frac{n-f}{n^2}$. An individual's effort contributed in the first (second) stage is decreasing in *f* (as long as $f \ge 2$). Thus, in order to maximize an individual's effort contributed in the two-stage tournament the organizers should minimize the number of participants in the second stage. The optimal architecture is to set $f = 2$ *i.e.* when the contestants are divided into two parallel sub-contests in the first stage and only two contestants compete for the final prize in the second stage.

In moderately discriminating contests (when $n/(n-1) < r \le 2$) some contestants may optimally drop out of competition either in the first or in the second stage of the two-stage tournament. Thus, the contest may end up in the asymmetric Nash equilibrium (*e.g.* the third row of table 1). The maximum number of active contestants in the first (second) stage is endogenously determined by the discriminating power of the contest success function and it is equal to $\text{int}\{r/(r-1)\}$ (*e.g.* Perez-Castrillo and Verdier 1992). The problem of the organizers who wish to maximize an individual's effort is then given by equation (2).[2]

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[2] The control variable *f* of the contests' organizers is drawn from the interval $[2, n]$. This enables a convenient comparison between the one-stage contest and the two-stage tournament within one optimization problem. When the solution of (2) is $f = n$ ($f \in [2, n]$) the one-stage contest (two-stage tournament) is optimal.

$$\max_{f\in[2,n]} \max\left\{ rV\frac{k-1}{k^2}, r\frac{V}{f}(1-r+r/k)\frac{l-1}{l^2} \right\}, \qquad (2)$$
$$k=\min\{f,\operatorname{int}\{r/(r-1)\}\}, l=\min\{n/f,\operatorname{int}\{r/(r-1)\}\}$$

In poorly discriminating contests (when $r \le n/(n-1)$) the number of active contestants in the first (second) stage $l = n/f$ ($k = f$) was exogenously determined by the organizers. In moderately discriminating contests (when $n/(n-1) < r \le 2$) the asymmetric Nash equilibria may arise endogenously in the two-stage tournament. The organizers face an additional constraint—they cannot assign more active contestants to a single stage of the tournament than the endogenous limit of $\operatorname{int}\{r/(r-1)\}$. This constraint is formalized in the second line of equation (2). The unique solution of problem (2) is $f = 2$ which coincides with the optimal architecture under the symmetric Nash equilibrium. Intuitively, the endogenous constraint on the maximum number of active contestants is not binding for the organizers who would always prefer to have a minimum number of active contestants in the second stage.

In highly discriminating contests (when $r > 2$) a Stackelberg equilibrium endogenously emerges in competition (*e.g.* Blavatskyy 2004). In the two-stage tournament only one contestant actively participates in the second stage and he or she contributes an effort $e_1 = V(r-1)^{1-1/r}/r$ (*e.g.* the fourth row of table 1). This effort does not depend on the contest architecture (the number f of the short-listed contestants). In the first stage also only one contestant actively participates in each divisional contest. He or she contributes an effort $\frac{V}{f}\left(1-\frac{(r-1)^{1-1/r}}{r}\right)$ which is always smaller than the effort e_1 contributed in the second stage. Therefore, when $r > 2$ the organizers have no control over maximum individual's effort contributed in the contest. Any contest architecture (either the one-stage contest or the two-stage tournament) would yield the same maximum individual's effort. The optimal contest architecture for maximizing an individual's effort (depending on the discriminating power of the contest success function) is summarized in the second column of table 2.

4.2. Maximum aggregate effort

Suppose that the organizers want to maximize the aggregate effort contributed in the two-stage tournament. In poorly discriminating contests (when $r \le n/(n-1)$) the symmetric Nash equilibrium exists in both stages. We already established that in the second stage *f* contestants then contribute effort $e_f = rV(f-1)/f^2$ each. In the first stage *n* contestants contribute effort $rV\left(1-r+\frac{r}{f}\right)\frac{n-f}{n^2}$ each. The problem of the organizers who want to maximize an aggregate effort is then given by equation (3).

$$\max_{f\in[2,n]}\left(rV\frac{f-1}{f}+rV\left(1-r+\frac{r}{f}\right)\frac{n-f}{n}\right) \qquad (3)$$

The straightforward maximization of problem (3) yields a solution $f=\sqrt{n}$ when $r<1$ and $f=n$ when $r>1$. In a special case when $r=1$ the aggregate effort contributed in the contest does not depend on the contest architecture (*e.g.* Gradstein and Konrad 1999). Intuitively, when the number *f* of the contestants who participate in the second stage increases, the aggregate effort contributed in the second stage increases but the aggregate effort contributed in the first stage decreases. When the contest is almost undiscriminating ($r<1$) these two effects can be optimally balanced if the number of finalists is $f=\sqrt{n}$.[3] When the contest is sufficiently discriminating ($r>1$) there is no interior solution. The aggregate effort is actually minimal when

[3] Gradstein and Konrad (1999) demonstrated that when $r<1$ the optimal architecture is a knock-out tournament (a series of pairwise contests). In other words, the optimal number of contestants in each stage should be minimum (two) and the number of stages should be maximum ($\mathrm{int}\{\log_2 n\}+1$). This paper, however, considers only the two-stage tournament. Because of this restriction the optimal number of contestants in each stage is $f=\sqrt{n}$.

$f=\sqrt{n}$ (*e.g.* Amegashie 1999). The aggregate effort can be maximized only when the number of finalists is either minimal ($f=2$) or maximal ($f=n$). And it turns out that in this case the aggregate effort is maximized when $f=n$ *i.e.* when the two-stage tournament collapses to the one-stage contest (*e.g.* Gradstein and Konrad 1999).

In moderately discriminating contests (when $n/(n-1)<r\le 2$) the organizers do not have full control over the number of active contestants in each stage due to the endogenous entry-exit in the asymmetric Nash equilibria. The maximum number of active contestants in each stage is endogenously restricted to be $\operatorname{int}\{r/(r-1)\}$. The problem of the aggregate effort maximization becomes (4).

$$\max_{f\in[2,n]}\left(rV\frac{k-1}{k}+rV\left(1-r+\frac{r}{k}\right)\frac{l-1}{l}\right), \qquad (4)$$
$$k=\min\{f,\operatorname{int}\{r/(r-1)\}\}, l=\min\{n/f,\operatorname{int}\{r/(r-1)\}\}$$

The solution of problem (4) is presented in appendix I. When $r\le\sqrt{n}/(\sqrt{n}-1)$ the optimal architecture is the two-stage tournament with $f=\operatorname{int}\{r/(r-1)\}$. When $r\ge\sqrt{n}/(\sqrt{n}-1)$ the optimal architecture is any two two-stage tournament with $\operatorname{int}\left\{\frac{r}{r-1}\right\}\le f\le\frac{n}{\operatorname{int}\left\{\frac{r}{r-1}\right\}}$.

Intuitively, the organizers who maximize the aggregate effort contributed in the contest would like to have as many active contestants in the second stage as possible. However, if the number f of the contestants who have the right to participate in the second stage exceeds $\operatorname{int}\{r/(r-1)\}$, some of them optimally choose to withdraw from competition in the second stage. Although such contestants are promoted to the second stage they contribute nothing to the aggregate effort in the second stage. From the point of view of aggregate efficiency too many contestants are promoted to the second stage: some of them should have been eliminated in the first stage. Therefore, the optimal architecture that maximizes the aggregate effort is the two-stage tournament. The number of finalists in the second stage $f = \text{int}\{r/(r-1)\}$ is such that if one more contestant is allowed to participate in the second stage he or she would optimally not use this right (by contributing zero effort in the second stage).

Finally, if the discriminating power of the contest success function is sufficiently high ($r \geq \sqrt{n}/(\sqrt{n}-1)$) the optimal contest architecture is not unique. The intuition is the following: when too many contestants are admitted to the second stage they contribute nothing to the aggregate effort in the second stage. However, if they participate only in the first stage they also optimally withdraw from the competition. Therefore, the precise regulation of the number of finalists in the second stage may not affect the aggregate effort contributed in the contest. Several contest architectures may yield the same aggregate effort.

As we have already established, in highly discriminating contests (when $r > 2$) only one contestant actively participates in the second stage of the two-stage tournament. He or she contributes effort $e_1 = V(r-1)^{1-1/r}/r$ (*e.g.* the fourth row of table 1). In the first stage also only one contestant actively participates in each of *f* divisional contests. He or she contributes effort $\frac{V}{f}\left(1-\frac{(r-1)^{1-1/r}}{r}\right)$. The aggregate effort contributed in the tournament is then *V*. The value of the prize is perfectly dissipated over the contributed aggregate effort. Any two-stage tournament would be the optimal architecture for maximizing the aggregate effort (the precise number of divisions is insignificant). The optimal contest architecture for maximizing the aggregate effort (depending on the discriminating power of the contest success function) is summarized in the third column of table 2.

4.3. Minimum standard deviation of effort

Suppose that the organizers want to minimize the standard deviation of effort contributed in the two-stage tournament. In poorly discriminating contests (when $r \le n/(n-1)$) the symmetric Nash equilibrium exists in both stages. In each stage every participating contestant contributes the same effort as the others. The standard deviation of contributed effort is zero (at its minimum). Therefore, any contest architecture yields minimum standard deviation of effort.

If the one-stage contest is organized when $n/(n-1) < r \le 2$ some contestants optimally withdraw from competition. In such asymmetric Nash equilibrium the standard deviation of effort is non-zero. The organizers can reduce the dispersion of effort by switching to the two-stage tournament. If the number f of the contestants who have the right to participate in the second stage is less than $\text{int}\{r/(r-1)\}$, all short-listed contestants actively participate in the second stage contest. The standard deviation of effort in the second stage is then zero. Additionally, if the number n/f of the contestants who participate in the first-stage divisional contest is less than $\text{int}\{r/(r-1)\}$, the standard deviation of effort in the first stage is then zero as well. Therefore, any two-stage tournament with $n\Big/\text{int}\left\{\frac{r}{r-1}\right\} \le f \le \text{int}\left\{\frac{r}{r-1}\right\}$ minimizes the standard deviation of contributed effort. Such a tournament is feasible when $\sqrt{n} \le \text{int}\{r/(r-1)\}$, which implies a restriction on the discriminating power of the contest success function $r \le \sqrt{n}/(\sqrt{n}-1)$.

When $r > \sqrt{n}/(\sqrt{n}-1)$ there is no two-stage tournament that can reduce the standard deviation of effort to zero in both stages. In the second stage the standard deviation of effort is $rV\frac{k-1}{k}\sqrt{\frac{1}{k}-\frac{1}{f}}$ (*e.g.* third row of table 1), where $k = \min\left\{f, \text{int}\left\{\frac{r}{r-1}\right\}\right\}$ is the number of active participants in the second stage. The standard deviation of effort contributed in the second stage strictly increases with f and falls to zero when $f \le \text{int}\{r/(r-1)\}$. In the first stage the standard deviation of effort is $r\frac{V}{f}\left(1-r+\frac{r}{k}\right)\frac{l-1}{l}\sqrt{\frac{1}{l}-\frac{f}{n}}$, where $l=\min\left\{\frac{n}{f},\text{int}\left\{\frac{r}{r-1}\right\}\right\}$ is the number of active participants in the first stage. The standard deviation of effort contributed in the first stage strictly decreases with f and falls to zero when $f \ge n \Big/ \text{int}\left\{\frac{r}{r-1}\right\}$. Therefore, the one-stage contest can never be optimal—any two-stage tournament with $f \ge n \Big/ \text{int}\left\{\frac{r}{r-1}\right\}$ has a lower standard deviation of effort in the second stage and zero standard deviation of effort in the first stage.

More generally, any two-stage tournament with $f < \text{int}\left\{\frac{r}{r-1}\right\}$ ($f > n \Big/ \text{int}\left\{\frac{r}{r-1}\right\}$) is correspondingly dominated by the two-stage tournament with $f = \text{int}\{r/(r-1)\}$ ($f = n \Big/ \text{int}\left\{\frac{r}{r-1}\right\}$). Additionally, the tournament with $f = \text{int}\left\{\frac{r}{r-1}\right\}$ has a lower standard deviation of effort in the first stage than the standard deviation of effort in the second stage of the tournament with $f = n \Big/ \text{int}\left\{\frac{r}{r-1}\right\}$ (both tournaments have zero standard deviation of effort correspondingly in the second and in the first stage). Finally, it turns out that the tournament with $f = \text{int}\{r/(r-1)\}$ is optimal because any tournament with higher f causes an increase in the standard deviation of effort contributed in the second stage, which is larger than overall standard deviation of effort contributed in the first stage (see proof in appendix II).

Intuitively, in the first stage the contestants compete for the expectation to win a prize in the second stage. Thus, the contributed effort and its standard deviation is much lower in the first stage than in the second stage, because in the second stage the contestants compete for an actual prize. Therefore, the cost of increasing the dispersion of effort in the second stage is much higher than in the first stage. Thus, the optimal contest architecture minimizes the standard deviation of effort in the second stage. The number of finalists $f = \text{int}\{r/(r-1)\}$ is such that if one more contestant is allowed to participate in the second stage he or she would optimally not use this right.

When $r > 2$ there is always only one contestant who actively participates in the contest. There is no two-stage tournament that can reduce the standard deviation of effort to zero in either stage. The standard deviation of effort contributed in the second stage is $V\frac{(r-1)^{1-1/r}}{r}\sqrt{1-1/f}$ (*e.g.* the fourth row of table 1). It strictly increases in *f* and reaches it's minimum when $f = 2$. The standard deviation of effort contributed in the first stage is $\frac{V}{f}\left(1-\frac{(r-1)^{1-1/r}}{r}\right)\sqrt{1-f/n}$. It strictly decreases in *f* and reaches its minimum in the one-stage contest (when $f = n$ and the first stage is omitted). The standard deviation of effort contributed in the first (second) stage of the two-stage tournament with $f = 2$ is $\frac{V}{2}\left(1-\frac{(r-1)^{1-1/r}}{r}\right)\sqrt{1-\frac{2}{n}}$ ($V\frac{(r-1)^{1-1/r}}{r\sqrt{2}}$) and it is lower than the standard deviation of effort contributed in the one-stage contest: $V\frac{(r-1)^{1-1/r}}{r}\sqrt{1-1/n}$. Therefore, the optimal architecture is the two-stage tournament with $f = 2$. The optimal contest architecture for minimizing the standard deviation of effort (depending on the discriminating power of the contest success function) is summarized in the fourth column of table 2.

<table>
<tr><th>Objective of the organizers
Discriminating power r of the contest success function</th><th>Maximum individual effort</th><th>Maximum aggregate effort</th><th>Minimum standard deviation of effort</th></tr>
<tr><td>$0 < r < 1$</td><td rowspan="5">Two-stage tournament
$f = 2$</td><td>Two-stage tournament
$f = \sqrt{n}$</td><td rowspan="3">Any</td></tr>
<tr><td>$r = 1$</td><td>Any</td></tr>
<tr><td>$1 < r \leq \frac{n}{n-1}$</td><td>One-stage contest $f = n$</td></tr>
<tr><td>$\frac{n}{n-1} < r \leq \frac{\sqrt{n}}{\sqrt{n}-1}$</td><td>Two-stage tournament
$f = \text{int}\left\{\frac{r}{r-1}\right\}$</td><td>Two-stage tournament
$\frac{n}{\text{int}\left\{\frac{r}{r-1}\right\}} \leq f \leq \text{int}\left\{\frac{r}{r-1}\right\}$</td></tr>
<tr><td>$\frac{\sqrt{n}}{\sqrt{n}-1} \leq r \leq 2$</td><td>Two-stage tournament
$\text{int}\left\{\frac{r}{r-1}\right\} \leq f \leq \frac{n}{\text{int}\left\{\frac{r}{r-1}\right\}}$</td><td>Two-stage tournament
$f = \text{int}\left\{\frac{r}{r-1}\right\}$</td></tr>
<tr><td>$r > 2$</td><td>Any</td><td>Any two-stage tournament</td><td>Two-stage tournament
$f = 2$</td></tr>
</table>

Table 2 The optimal architecture of imperfectly discriminating contests with *n* contestants (in the two-stage tournament *f* short-listed contestants compete in the second stage)

5. Conclusion

This paper explains an empirical observation that the one-stage contests are rarely organized in real life. The majority of real life competitions are organized instead as the two-stage tournaments. In the first stage the contestants compete in several parallel contests for the right to participate in the second stage. In the second stage the short-listed contestants compete for the final prize. For example, at the Olympics there is typically a qualification stage and a final stage. One can think of other examples of multi-stage contests, such as job interviews, political elections etc.

The main theoretical result of this paper is the following. In symmetric imperfectly discriminating contests the optimal contest architecture is generically the two-stage tournament. Given a sufficient number of contestants, the two-stage tournament is either strictly better or at least as good as the one-stage contest for maximizing an individual's effort, for maximizing the aggregate effort and for minimizing the standard deviation of effort. Only on one instance when the discriminating power r of the contest success function is between unity and $n/(n-1)$ the one-stage contest is an optimal architecture for maximizing the aggregate effort. However, when the number of contestants is very large the probability that $r \in (1, n/(n-1)]$ converges to zero.

When the organizers want to maximize an individual's effort the optimal architecture is generically the two-stage tournament with only two contestants in the second stage. Moldovanu and Sela (2004) obtained the same result for perfectly discriminating contests. When the organizers want to maximize the aggregate effort contributed in the contest or to minimize the standard deviation of effort, the optimal number of finalists in the second stage depends on the discriminating power of the contest success function. In moderately discriminating contests (when $n/(n-1) < r \leq 2$) the optimal architecture for maximizing the aggregate effort (minimizing the standard deviation of effort) is generically the following. The two-stage tournament should be organized with such a number of finalists in the second stage that if one more contestant is allowed to participate in the second stage he or she would optimally not use this right.

Interestingly, in highly discriminating contests (when $r > 2$) the optimal architecture for maximizing the aggregate effort is any two-stage tournament. It does not matter how many finalists are admitted to the second stage. Only the fact that there are two stages in the tournament is sufficient to maximize the aggregate effort contributed in the competition. This result contrasts with the finding in Moldovanu and Sela (2004) that the one-stage contest is optimal for maximizing the aggregate effort in perfectly discriminating contests (that may be thought of as a limiting case of highly discriminating contests).

In this paper the analysis was restricted to the tournament that has only two stages, which is common in the literature on contest architecture (*e.g.* Moldovanu and Sela 2004, Amegashie 1999). A natural extension of this work is to make the number of stages in the tournament a policy variable for the organizers. It would be interesting to explore under which conditions the multi-stage tournaments are preferred to the two-stage tournament and the one-stage contest. In the two-stage tournament, as modeled in this paper, only one contestant was promoted from each first stage division to the second stage. However, in many real life competitions several top ranked contestants are promoted from each sub-division to the next stage. It would be interesting to explore if the promotion of several top ranked contestants is optimal.

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Appendix I

The solution of the aggregate effort maximization problem (4).

Three cases are possible.

a) When $f \geq \text{int}\left\{\frac{r}{r-1}\right\}$ and $\frac{n}{f} \leq \text{int}\left\{\frac{r}{r-1}\right\}$ problem (4) simplifies into (A1).

$$\max_{f\in[2,n]}\left(rV\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}} + rV\left(1-r+\frac{r}{\text{int}\left\{\frac{r}{r-1}\right\}}\right)\frac{n-f}{n}\right) \qquad \text{(A1)}$$

The objective function in (A1) is decreasing in *f*. Therefore, it is optimal to set *f* to its minimum possible value: $f = \text{int}\left\{\frac{r}{r-1}\right\}$. Since case a) is possible only when $\frac{n}{f} \leq \text{int}\left\{\frac{r}{r-1}\right\}$, the solution $f = \text{int}\left\{\frac{r}{r-1}\right\}$ applies only when $\sqrt{n} \leq \text{int}\left\{\frac{r}{r-1}\right\}$. The last inequality implies a restriction on the discriminating power of the contest success function $r \leq \frac{\sqrt{n}}{\sqrt{n}-1}$.

b) When $f \leq \text{int}\left\{\frac{r}{r-1}\right\}$ and $\frac{n}{f} \geq \text{int}\left\{\frac{r}{r-1}\right\}$ problem (4) simplifies into (A2).

$$\max_{f\in[2,n]}\left(rV\frac{f-1}{f} + rV\left(1-r+\frac{r}{f}\right)\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}}\right) \qquad \text{(A2)}$$

The objective function in (A2) can be rearranged into (A3).

$$\max_{f\in[2,n]}\left(rV + rV(1-r)\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}} + \frac{rV}{f}\underbrace{\left(r\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}}-1\right)}_{\leq 0}\right) \qquad \text{(A3)}$$

The objective function in (A3) is increasing in *f*. Therefore, it is optimal to set *f* to its maximum possible value: $f = \text{int}\left\{\frac{r}{r-1}\right\}$.

c) When $f \geq \text{int}\left\{\frac{r}{r-1}\right\}$ and $\frac{n}{f} \geq \text{int}\left\{\frac{r}{r-1}\right\}$ problem (4) simplifies into (A4).

$$\max_{f\in[2,n]}\left(rV\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}} + rV\left(1-r+\frac{r}{\text{int}\left\{\frac{r}{r-1}\right\}}\right)\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}}\right) \qquad \text{(A4)}$$

The objective function in (A4) does not depend on *f*. Therefore, any contest architecture yields the same aggregate effort. This case happens when $\frac{n}{\text{int}\left\{\frac{r}{r-1}\right\}} \geq f \geq \text{int}\left\{\frac{r}{r-1}\right\}$ which implies a restriction on the discriminating power of the contest success function $r \geq \frac{\sqrt{n}}{\sqrt{n}-1}$.

Appendix II

Proof that the two-stage tournament with $f = \text{int}\left\{\frac{r}{r-1}\right\}$ is optimal for minimizing the standard deviation of effort when $\frac{\sqrt{n}}{\sqrt{n}-1} \le r \le 2$. We already know that the optimal architecture must be a two-stage tournament with $\text{int}\left\{\frac{r}{r-1}\right\} \le f \le \frac{n}{\text{int}\left\{\frac{r}{r-1}\right\}}$ (all other tournaments and the one-stage contest are strictly dominated—they yield a higher standard deviation of effort at least in one stage).

When $\text{int}\left\{\frac{r}{r-1}\right\} \le f \le \frac{n}{\text{int}\left\{\frac{r}{r-1}\right\}}$, the standard deviation of effort contributed in the first stage ranges from zero (when $f = n \Big/ \text{int}\left\{\frac{r}{r-1}\right\}$) up to

$$S_1 = r\frac{V}{\text{int}\left\{\frac{r}{r-1}\right\}}\underbrace{\left(1-r+\frac{r}{\text{int}\left\{\frac{r}{r-1}\right\}}\right)}_{\le 1}\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}}\sqrt{\frac{1}{\text{int}\left\{\frac{r}{r-1}\right\}}-\frac{\text{int}\left\{\frac{r}{r-1}\right\}}{n}} \quad \text{(when } f = \text{int}\left\{\frac{r}{r-1}\right\}\text{)}.$$

Since $\text{int}\left\{\frac{r}{r-1}\right\} \le \frac{r}{r-1}$ then it follows that $1-r+\frac{r}{\text{int}\left\{\frac{r}{r-1}\right\}} \le (r-1)^2 \le 1$ with the last inequality due to $r \le 2$. Thus, $S_1 \le rV\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\left(\text{int}\left\{\frac{r}{r-1}\right\}\right)^{2.5}}$.

The standard deviation of effort contributed in the second stage is zero when $f=\text{int}\left\{\frac{r}{r-1}\right\}$ and it strictly increases in *f*. When $f=\text{int}\left\{\frac{r}{r-1}\right\}+1$ it is already

$$S_2=rV\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\text{int}\left\{\frac{r}{r-1}\right\}}\sqrt{\frac{1}{\text{int}\left\{\frac{r}{r-1}\right\}}-\frac{1}{\text{int}\left\{\frac{r}{r-1}\right\}+1}}=rV\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\left(\text{int}\left\{\frac{r}{r-1}\right\}\right)^{1.5}\sqrt{\text{int}\left\{\frac{r}{r-1}\right\}+1}}>rV\frac{\text{int}\left\{\frac{r}{r-1}\right\}-1}{\left(\text{int}\left\{\frac{r}{r-1}\right\}\right)^{2.5}}$$

. The last inequality holds because $\text{int}\left\{\frac{r}{r-1}\right\}>\frac{\sqrt{5}+1}{2}$ when $r\le 2$.

We demonstrated that $S_2>S_1$ *i.e.* the minimum non-zero standard deviation of effort contributed in the second stage always exceeds the maximum possible standard deviation of effort contributed in the first stage. In other words there is no trade-off in this case. When *f* increases the standard deviation of effort contributed in the first (second) stage decreases (increases). However, any increase in the standard deviation of effort contributed in the second stage is always much higher than any decrease in the standard deviation of effort contributed in the first stage. Thus, the optimal architecture should set the standard deviation of effort contributed in the second stage to zero. In other words, the optimal architecture is the two-stage tournament with $f=\text{int}\left\{\frac{r}{r-1}\right\}$.